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POSITIVE PRESENTATIONS OF FAMILIES RELATIVE TO e-ORACLES

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Abstract: We introduce the notion of A-numbering which generalizes the classical notion of numbering. All main attributes of classical numberings are carried over to the objects considered here. The problem is investigated of the existence of positive and decidable computable A-numberings for the natural families of sets e-reducible to a fixed set. We prove that, for every computable A-family containing an inclusion-greatest set, there also exists a positive computable A-numbering. Furthermore, for certain families we construct a decidable (and even single-valued) computable total A-numbering when A is a low set; we also consider a relativization containing all cases of total sets (this in fact corresponds to computability with a usual oracle).

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The paper studies the objects that are generalizations of the numberings in [1] and some particular variants of the $\mathbb{A}$-numberings (here $\mathbb{A}$ is a suitable admissible set) introduced in [2] (in view of the existence of the transformation effecting a transition from e-degrees to admissible sets [3]). The key problem to be addressed is the existence of the Friedberg (single-valued computable) and positive presentations of families. It was shown in [3] that the above transformation preserves most of the considered properties of descriptive set theory. However, it is not hard to demonstrate that the transformation also preserves positive (negative, decidable, and single-valued) presentations. We focus the reader's attention on the fact that we will have to extend the notion of numbering and, in the general case, consider not total but partial mappings. This effect arises in passage from a hereditarily finite superstructure to natural numbers since for nontotal sets a computable function (in the sense of the hereditarily finite superstructure) that enumerates the hereditarily finite superstructure by means of natural numbers is necessarily a partial function.

We prove that the representatives of a wide class of families have positive computable presentation.

The main information about classical computability can be found in [4]; about numbering theory, in [1], and from the theory of admissible sets, in [2]. Firstly, we introduce the main notions and notations of use in the article.

Denote by $\leftrightharpoons$ equality by definition. The set of naturals will be denoted by ω. The symbol c stands for the Cantor numbering of the pairs of naturals.

If R is a binary relation and f is a function of an arbitrary nature then we denote by δR and δf the projections to the first coordinate of the relation and to the domain of the function respectively and designate as ρR and ρf the projections to the second coordinate and the range respectively.

Given $A, B \subseteq \omega$, we say that A is *e-reducible* to B ($A \leq_e B$ in notation) if there exists a binary computably enumerable (c.e.) predicate W for which $A = \{n \mid \exists u[\langle n, u\rangle \in W \wedge \gamma(u) \subseteq B]\}$, where $\gamma(u)$ is the finite set with canonical number u, i.e. the set of naturals F such that $\sum_{i\in F} 2^i = u$. In the literature, the e-reducibility is also called *enumeration reducibility* (see [5]).

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